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Backward Integration of the NMC 8-Level Global Model

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### 1. Introduction

One of the problems in four-dimensional data assimilation is the "rejection" phenomenon: the tendency of a numerical prediction model to ignore inserted asynoptic observations, thus restoring the dynamic balance existing prior to insertion. The effect is to reduce the influence of the inserted observations. Morel, Lefevre and Rabreau (1971) have suggested repeatedly introducing each observation as the model is integrated forward and backward over the interval containing the data as a means of overcoming this problem.

A requirement for the successful use of this cycling technique is that the model be reversible in time, in the absence of any inserted data. Clearly, if model errors grow as a function of the number of time steps, without regard to the direction of integration, the technique would not be useful. The vehicle used by Morel, et. al., a primitive equation barotropic model, is essentially reversible, and their experiment showed promising results.

The application of this technique to a multilevel model immediately raises the question of reversibility. This note describes a set of experiments designed to examine the degree to which the NMC 8L GLOBAL model is reversible.

### 2. Procedure

The version of the model used in these experiments has  $5^{\circ}$  resolution in the horizontal and 8 meteorologically active layers in the vertical. All modeling of irreversible physical processes was removed, including precipitation, radiation, sensible heat exchange, and friction. The dry convective adjustment, while irreversible, was retained as a necessary numerical control. Three experiments were performed:

1. model stripped except for the dry convective adjustment;
2. the Robert time filter (coefficient of 0.5) added to (1);
3. a special viscosity term (the "divergence damper," coefficient of  $10^7 \text{ m}^2 \text{ sec}^{-1}$ ; see NMC Office Note 83) added to (2).

The first experiment was simply to determine the maximum degree of reversibility attainable. The second and third were directed at the reversibility question if the damping integration methods necessary for four-dimensional assimilation are included.

Each experiment began from 1200 GMT 6 September 1974, using an initial state provided by the spectral global analysis model, without further initialization. The integrations marched forward in steps of 20 minutes for 12 hours, when time was reversed. Integration then proceeded backward to the initial time, producing a "final" state.

### 3. Results

Root-mean-square (RMS) differences between initial and final states by sigma-level for the history variables,  $T$ ,  $u$ ,  $v$ ,  $P_0$ , and  $q$ , as well as for divergence ( $D$ ) and vorticity ( $\zeta$ ), are given in Table 1. For the stripped experiment, the RMS differences are small. The greatest differences in the history variables in the lowest sigma layers, presumably due to dry convective adjustment. Divergence has been changed, on a percentage basis, substantially more than the vorticity.

The Robert time filter has the most noticeable effect on the pressure-thickness field. Removal of high-frequency gravity wave noise by the filter results in the larger RMS  $P_0$  differences. This is also reflected in the larger RMS divergence differences while the RMS vorticity differences are changed only slightly.

Adding the divergence damper (Table 1c) results in much larger RMS differences in every quantity except moisture, which is evidently affected mostly by the convective adjustment.

Individual values of surface and tropopause pressures for two grid points are given in Tables 2-4 for each hour during forward and backward integration. One grid point, 35N, 70W, is off the east coast of the United States; the other, 35S, 70W, is over the Andes. Experiment 1, which excludes both time filter and divergence damper, is shown in Table 2. The maximum difference is 0.3 mb.

The behavior of the surface pressure at both points is characterized by high-frequency gravity wave noise. In Figure 1, the hourly values of surface pressure at the Southern Hemisphere point are plotted for the backward part of the integration. Adding the time filter (experiment 2) damps the noise, as does the addition of the divergence damper (experiment 3). The hourly values for experiments 2 and 3 are given in Tables 3 and 4. Evidently, the differences between forward and backward integration are largely due to the suppression of the noise.

RMS surface pressure tendencies at hourly intervals are presented in Table 5. Without the time filter and divergence damper, the RMS tendency is 0.24 mb/time step initially and increases to about 1 mb per time step (3 mb/hr) by the 12th hour. The backward integration produces the same tendencies, again illustrating the high degree of

reversibility of the stripped model. In the second experiment, there is an initial increase during the first hour, followed by a steady decline throughout the forward and backward integration. The final value is about 0.3 mb/hr. Adding the divergence damper does not contribute very much more to the suppression of the noise, as measured by the RMS surface pressure tendency.

#### 4. Conclusions

These experiments indicate that the 8-level model, stripped of physical parameterizations, can be successfully integrated backward in time. Incorporation of the Robert time filter still results in a high degree of reversibility, and exhibits considerable capacity for noise suppression. This version of the model therefore appears to be suitable for four-dimensional data assimilation experiments within the cycling framework proposed by Morel, et. al.

#### REFERENCE

- Morel, P., G. Lefevre, and G. Rabreau, 1971: On initialization and non-synoptic data assimilation. Tellus, vol. 23, no. 3.

Table 1. RMS differences between initial values and final values of several parameters, after marching forward 12 hours and then backward to the initial time. Parenthetical entries for divergence and vorticity express the ratio of the RMS difference to the RMS of the initial value, in percent.

a. excluding both time filter and divergence damper

Parameter	$\sigma$ -level								
	1	2	3	4	5	6	7	8	9
T (deg)	0.91	0.59	0.46	0.33	0.16	0.07	0.07	0.06	0.00
u (m sec <sup>-1</sup> )	0.73	0.77	0.47	0.59	0.36	0.20	0.11	0.10	0.03
v (m sec <sup>-1</sup> )	0.66	0.63	0.49	0.49	0.40	0.21	0.11	0.12	0.04
P <sub>σ</sub> (mb)	0.23	-----	-----	-----	-----	-----	0.17	-----	0.02
D (10 <sup>6</sup> sec <sup>-1</sup> )	1.67	1.51	1.11	1.24	0.89	0.58	0.33	0.36	0.10
	(55)	(48)	(34)	(34)	(29)	(16)	(10)	(23)	
ζ (10 <sup>6</sup> sec <sup>-1</sup> )	1.66	1.67	1.27	1.35	1.01	0.41	0.23	0.16	0.93
	(13)	(11)	(907)	(06)	(04)	(01)	(01)	(01)	
q (gm/kgm)	0.12	0.06	0.03	0.03	0.03	-----	-----	-----	-----

b. including the time filter, but excluding the divergence damper

Parameter	1	2	3	4	5	6	7	8	9
T	0.91	0.58	0.48	0.32	0.18	0.16	0.29	0.25	0.00
u	0.78	0.73	0.56	0.63	0.47	0.53	0.46	0.82	0.64
v	0.72	0.68	0.54	0.56	0.51	0.50	0.39	0.57	0.34
P <sub>σ</sub>	1.60	-----	-----	-----	-----	-----	0.95	-----	0.31
D	1.85	1.71	1.34	1.34	1.24	1.15	1.01	1.00	0.53
	(61)	(55)	(40)	(38)	(40)	(32)	(31)	(64)	
ζ	1.74	1.63	1.35	1.35	1.12	0.94	0.55	1.13	0.60
	(13)	(11)	(08)	(06)	(04)	(03)	(02)	(07)	
q	0.12	0.06	0.03	0.03	0.03	-----	-----	-----	-----

c. including both time filter and divergence damper

Parameter	1	2	3	4	5	6	7	8	9
T	1.11	0.84	0.72	0.66	1.00	0.90	2.42	2.32	0.00
u	1.94	1.14	1.06	1.14	1.30	1.99	1.59	1.82	0.65
v	1.67	1.17	1.26	1.12	1.58	1.98	1.82	2.11	0.59
P <sub>σ</sub>	3.97	-----	-----	-----	-----	-----	3.50	-----	0.49
D	4.42	2.78	2.96	2.92	3.78	4.09	4.66	4.56	1.26
	(147)	(89)	(89)	(82)	(124)	(113)	(140)	(291)	
ζ	5.36	3.22	3.33	3.20	3.85	5.39	4.52	4.48	.93
	(41)	(22)	(19)	(15)	(14)	(18)	(19)	(28)	
q	0.12	0.06	0.03	0.03	0.04	-----	-----	-----	-----

Table 2. Hourly values of surface and tropopause pressures at two grid points during forward and backward integration. Time filter and divergence damper excluded. Values marked by asterisks indicate differences of as much as 0.1 mb.

HOUR	35N, 70W				35S, 70W			
	$P_s$		$P_T$		$P_s$		$P_T$	
	FORWARD	BACKWARD	FORWARD	BACKWARD	FORWARD	BACKWARD	FORWARD	BACKWARD
0	1019.7	1019.6*	115.3	115.2*	753.8	753.8	189.5	189.8*
1	1019.0	1018.9*	115.6	115.6	758.2	758.2	191.2	191.4*
2	1020.1	1020.0*	116.1	116.1	762.5	762.5	192.5	192.6*
3	1019.0	1019.0	115.9	115.9	760.2	760.1*	192.6	192.7*
4	1018.1	1018.1	115.7	115.7	757.5	757.5	192.9	192.9
5	1019.6	1019.6	115.9	115.9	758.9	758.8*	193.9	194.0*
6	1018.3	1018.3	115.7	115.7	757.3	757.2*	194.1	194.1
7	1015.9	1015.9	115.3	115.3	754.3	754.2*	193.6	193.6
8	1015.8	1015.8	115.3	115.3	756.6	756.5	193.9	193.9
9	1016.8	1016.8	115.6	115.6	757.2	757.2	193.3	193.3
10	1019.8	1019.8	116.3	116.3	753.8	753.9*	191.3	191.3
11	1018.8	1018.8	116.4	116.4	754.7	754.7	189.8	189.8
12	1014.3	1014.3	115.7	115.7	756.5	756.5	188.0	188.0

Table 3. Hourly values of surface and tropopause pressures of two grid points during forward and backward integration. Time filter included, divergence damper excluded.

HOUR	35N, 70W				35S, 70W			
	$P_s$		$P_T$		$P_s$		$P_T$	
	FORWARD	BACKWARD	FORWARD	BACKWARD	FORWARD	BACKWARD	FORWARD	BACKWARD
0	1019.7	1019.8	115.3	115.4	753.8	757.4	189.5	189.6
1	1019.0	1019.5	115.6	115.6	758.6	758.3	191.2	190.8
2	1019.9	1019.3	116.0	115.8	762.0	759.1	192.4	191.8
3	1018.6	1019.1	115.8	115.8	759.6	759.2	192.5	192.6
4	1018.9	1018.8	115.8	115.8	758.6	758.6	193.1	193.2
5	1019.0	1018.4	115.8	115.7	758.3	757.8	193.8	193.6
6	1017.9	1017.7	115.6	115.6	756.5	757.0	193.8	193.8
7	1016.4	1017.0	115.4	115.5	756.3	756.4	193.9	193.7
8	1016.4	1017.0	115.4	115.6	756.4	756.1	193.6	193.4
9	1018.0	1017.6	115.8	115.8	755.5	755.6	192.7	192.6
10	1017.8	1017.7	116.0	116.0	755.2	755.3	191.5	191.3
11	1017.3	1017.4	116.1	116.1	755.2	755.2	189.7	189.6
12	1018.1	1018.1	116.3	116.3	755.2	755.5	187.4	187.3

Table 4. Hourly values of surface and tropopause pressures at two grid points during forward and backward integration. Time filter and divergence damper included.

HOUR	35N, 70W				35S, 70W			
	$P_s$		$P_T$		$P_s$		$P_T$	
	FORWARD	BACKWARD	FORWARD	BACKWARD	FORWARD	BACKWARD	FORWARD	BACKWARD
0	1019.7	1019.4	115.3	116.3	753.8	756.9	189.5	189.9
1	1019.0	1018.8	115.6	116.1	758.8	758.0	191.2	190.8
2	1020.0	1018.4	116.0	115.9	761.9	759.1	192.6	191.8
3	1018.5	1018.1	115.8	115.8	759.8	759.8	192.4	192.9
4	1019.1	1018.0	115.9	115.6	758.6	759.6	192.8	193.9
5	1019.0	1017.6	115.8	115.3	758.0	758.6	193.3	194.4
6	1017.8	1016.9	115.6	115.1	756.5	758.1	193.2	194.5
7	1016.4	1016.5	115.3	115.0	756.4	757.8	193.1	193.9
8	1016.6	1017.0	115.3	115.1	756.6	757.1	192.5	192.7
9	1018.2	1017.4	115.6	115.3	755.9	756.3	191.3	191.2
10	1017.6	1017.5	115.5	115.4	755.3	755.7	189.6	189.5
11	1017.7	1017.9	115.5	115.5	755.2	755.3	187.5	187.4
12	1018.5	1018.4	115.6	115.6	755.4	755.4	184.9	184.8



Table 5. RMS surface pressure tendency in mb/time step during forward and backward integration, for experiments 1-3.

HOUR	EXPERIMENT 1		EXPERIMENT 2		EXPERIMENT 3	
	FORWARD	BACKWARD	FORWARD	BACKWARD	FORWARD	BACKWARD
0	0.24	0.24	0.24	0.11	0.24	0.11
1	0.62	0.62	0.64	0.13	0.63	0.12
2	0.65	0.65	0.52	0.15	0.51	0.14
3	0.58	0.58	0.40	0.15	0.40	0.14
4	0.65	0.65	0.37	0.15	0.37	0.14
5	0.62	0.62	0.35	0.15	0.36	0.14
6	0.69	0.69	0.32	0.15	0.33	0.14
7	0.73	0.73	0.31	0.17	0.32	0.17
8	0.81	0.81	0.30	0.19	0.31	0.18
9	0.80	0.80	0.27	0.18	0.27	0.18
10	0.92	0.92	0.24	0.19	0.27	0.19
11	0.88	0.88	0.24	0.21	0.26	0.23
12	1.04	1.04	0.23	0.22	0.24	0.22

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$P_s$   
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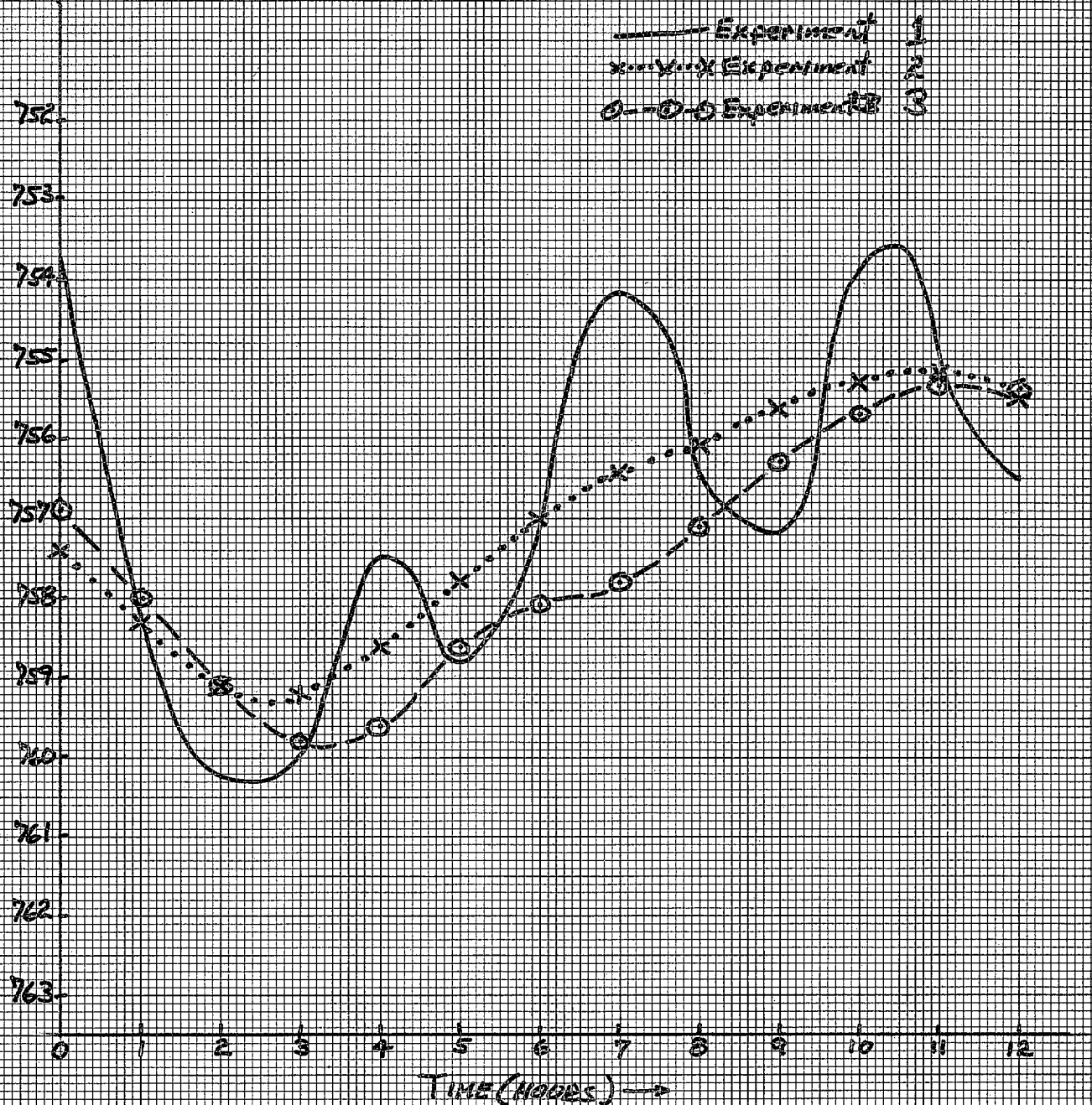


Figure 1. Surface pressure as a function of time during backward integration at a point 35S, 70W. See text for explanation of each experiment.